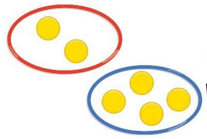
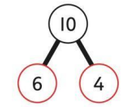
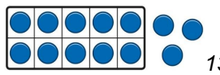
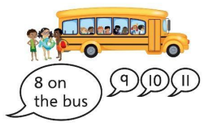
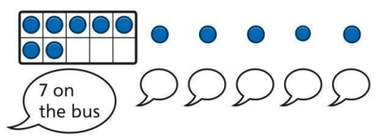
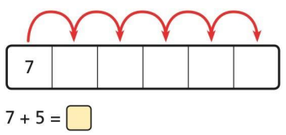
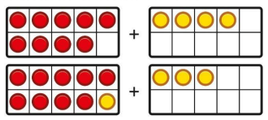
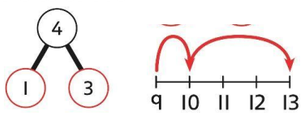
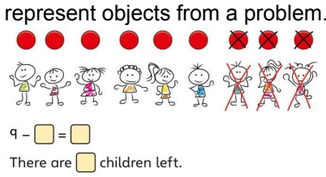
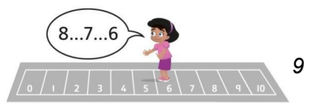
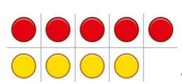
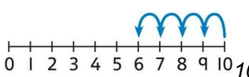
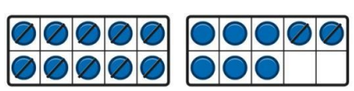
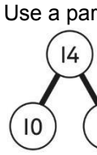
	End points	Concrete	Pictorial	Abstract	Language
YEAR 1	• Compose numbers to 10 from 2 parts, and partition numbers to 10 into parts, including recognising odd and even numbers. • Read, write and interpret equations containing addition, subtraction and equals symbols, and relate additive expressions and equations to real-life contexts. • add and subtract one-digit and two-digit numbers to 20, including 0 • solve one-step problems that involve addition and subtraction, using concrete objects and pictorial representations, and missing number problems • Develop fluency in addition and subtraction facts within 10.	Sort people and objects into parts and understand the relationship with the whole.  *The parts are 2 and 4. The whole is 6.*	Children draw to represent the parts and understand the relationship with the whole.  *The parts are 1 and 5. The whole is 6.*	Use a part-whole model to represent the numbers.  *6 + 4 = 10 10* 6 + 4 = 10	whole, part, ones, tens, number bond, add, addition, plus, total, altogether, subtract, subtraction, find the difference, take away, minus **Addition** is putting the **parts** together. That makes the **whole.** When you add, the **whole** is always on its own. Bonds to 10: …. and…… make 10. 10 is …. and …… **Subtraction** is taking away a part from the whole. That leaves the other part. When you subtract, the **whole** is before the symbol.
		Complete a group of 10 objects and count more.  *13 is 10 and 3 more.*	Use a ten frame to support understanding of a complete 10 for teen numbers.  *13 is 10 and 3 more.*	*1 ten and 3 ones equal 13.* *10 + 3 = 13*	
		Children use knowledge of counting to 20 to find a total by counting on using people or objects.  	Children use counters to support and represent their counting on strategy.  	Children use number lines or number tracks to support their counting on strategy.  	
		Children use a bead string to complete a 10 and understand how this relates to the addition.  *7 add 3 makes 10.* *So, 7 add 5 is 10 and 2 more.*	Children use counters to complete a ten frame and understand how they can add using knowledge of number bonds to 10. 	Use a part-whole model and a number line to support the calculation.  	
		Counting back and taking away Children arrange objects and remove to find how many are left.  *1 less than 6 is 5.* *6 subtract 1 is 5.*	Children draw and cross out or use counters to represent objects from a problem.  	Children count back to take away and use a number line or number track to support the method.  *9 − 3 = 6*	
		Arrange two groups so that the difference between the groups can be worked out.  *8 is 2 more than 6.* *6 is 2 less than 8.* *The difference between 8 and 6 is 2.*	Represent objects using sketches or counters to support finding the difference.  *5 − 4 = 1* *The difference between 5 and 4 is 1.*	Children understand ‘find the difference’ as subtraction.  *10 − 4 = 6* *The difference between 10 and 6 is 4.*	
		Subtract 12 by first subtracting the 10, then the remaining 2.  *First subtract the 10, then take away 2.*	Use ten frames to represent the efficient method of subtracting 12.  *First subtract the 10, then subtract 2.*	Use a part-whole model to support the calculation.  *19 − 14* *19 − 10 = 9* *9 − 4 = 5* *So, 19 − 14 = 5*	

YEAR 2

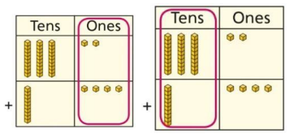
- Secure fluency in addition and subtraction facts within 10, through continued practice.
- Add and subtract across 10
- Recognise the subtraction structure of 'difference' and answer questions of the form, "How many more…?".
- Add and subtract within 100 by applying related one digit addition and subtraction facts: add and subtract only ones or only tens to/from a two-digit number.
- Add and subtract within 100 by applying related one digit addition and subtraction facts: add and subtract any 2 two digit numbers.

Use known bonds and unitising to add 10s.

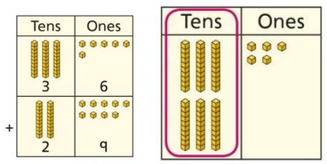
I know that 4 + 3 = 7.
So, I know that 4 tens add 3 tens is 7 tens.

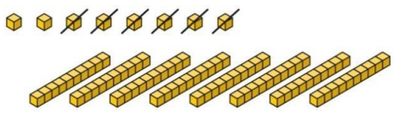
There are 4 tens and 5 ones.
I need to add 7. I will use 5 to complete a 10, then add 2 more.

Use dienes to Add the 1s. Then add the 10s.

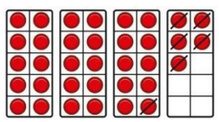


Use dienes to Add the 1s. Exchange 10 ones for a ten. Then add the 10s.

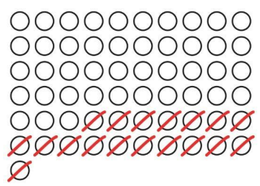




8 subtract 6 is 2.
So, 8 tens subtract 6 tens is 2 tens.

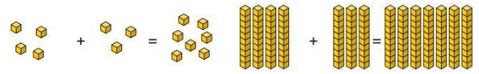


35 − 6
I took away 5 counters, then 1 more.

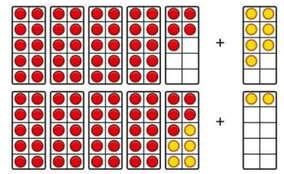


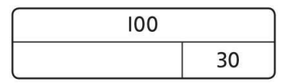
61 − 18
I took away 1 ten and 8 ones.

Use known bonds and unitising to add 10s.

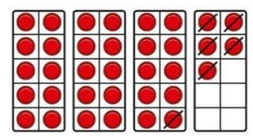


I know that 4 + 3 = 7.
So, I know that 4 tens add 3 tens is 7 tens.

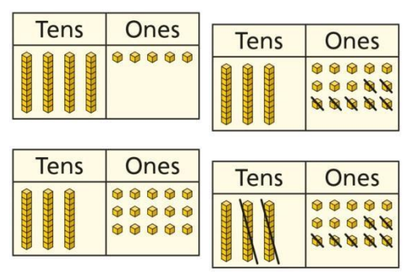




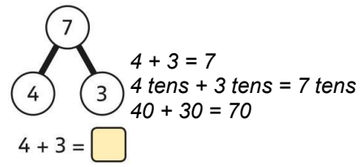
10 − 3 = 7
So, 10 tens subtract 3 tens is 7 tens.



35 − 6
First, I will subtract 5, then 1.

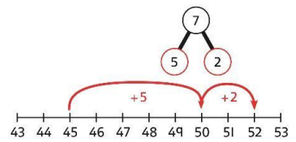


Use known bonds and unitising to add 10s.

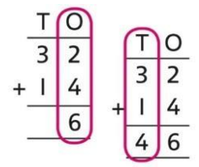


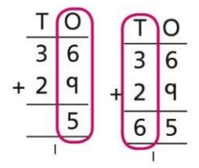
4 + 3 = 7
4 tens + 3 tens = 7 tens
40 + 30 = 70

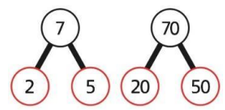
4 + 3 = ☐



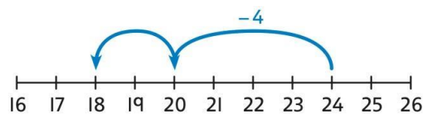
7 = 5 + 2
45 + 5 + 2 = 52



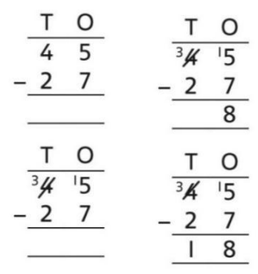




7 tens subtract 5 tens is 2 tens.
70 − 50 = 20



24 − 6 = ?
24 − 4 − 2 = ?



whole, part, ones, tens, number bond, add, addition, plus, total, altogether, subtract, subtraction, find the difference, take away, minus**, exchange**

Concept
Addition is putting together the parts,that makes the whole**.**

Addition is commutative
Commutative means we can switch the parts around and the whole stays the same.

When you add, the **whole** is always on its own.

Bonds to 10:
…. and…… make 10. 10 is …. and ……

E.g 17 + 5 = 22
22 = 17 + 5

Strategy

Column Method:
1st you add the ones, then you add the tens

Exchanging when you're adding:
10 ones make one 10. 10 tens make 100.

Concept
Subtraction is taking away a part from the whole. That leaves the other part.

When you subtract, the **whole** is before the subtraction symbol.

Strategy

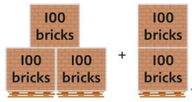
When the **whole and part are far apart, take away the part.**
When the whole and part are close together, count on from the part

Column subtraction:
1st you subtract the ones, then you subtract the tens.

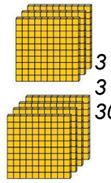
Exchanging when you're subtracting:
Exchange 1 ten for 10 ones.

YEAR 3

- Calculate complements to 100
- Add and subtract up to three-digit numbers using columnar methods.
- Manipulate the additive relationship: Understand the inverse relationship between addition and subtraction, and how both relate to the part–part–whole structure. Understand and use the commutative property of addition
- Secure fluency in addition and subtraction facts that bridge 10, through continued practice.



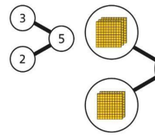
3 + 2 = 5
3 hundreds + 2 hundreds = 5 hundreds
300 + 200 = 500



3 + 4 = 7
3 hundreds + 4 hundreds = 7 hundreds
300 + 400 = 700

Represent the addition on a number line.

Use a part-whole model to support unitising.



3 + 2 = 5
300 + 200 = 500

partition, place value, tens, hundreds, thousands, column method, whole, part, bar model

The whole is always the biggest.

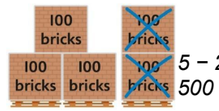
Column Addition
First we add the ones, then we do the tens.

First, we add the ones and carry over.
Next, we add the tens and carry over.
Then we add the hundreds and carry over.

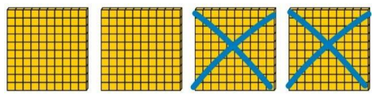
When you add the answer gets bigger
I exchange ten ones for one ten.

I exchange ten tens for one hundred.

The whole is always the biggest.



5 − 2 = 3
500 − 200 = 300

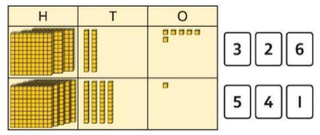


4 − 2 = 2
400 − 200 = 200

400 − 200 = 200

Use known facts and unitising as efficient and accurate methods.
I know that 7 − 4 = 3. Therefore, I know that 700 − 400 = 300.

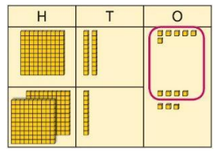
326 + 541 is represented as:



Represent the place value grid with equipment to model the stages of column addition.

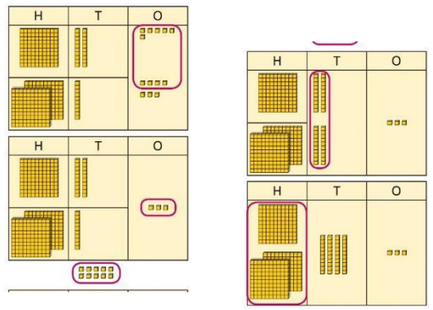
Use a column method to solve efficiently, using known bonds. Children must understand how this relates to place value at every stage of the calculation.

Use place value equipment to enact the exchange required.

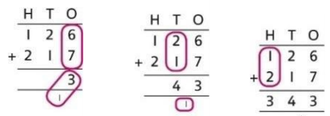


There are 13 ones.
I will exchange 10 ones for 1 ten.

Model the stages of column addition using place value equipment on a place value grid.

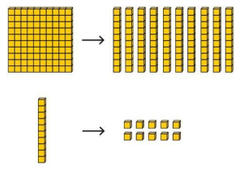


Use column addition, ensuring understanding of place value at every stage of the calculation.

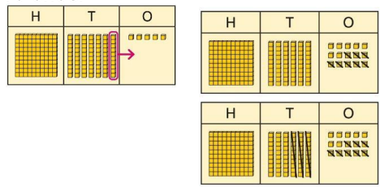


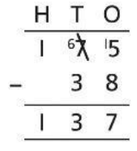
126 + 217 = 343
Note: Children should also study examples where exchange is required in more than one column, for example *185 + 318 = ?*

Use equipment to enact the exchange of 1 hundred for 10 tens, and 1 ten for 10 ones.



175 − 38 = ?
I need to subtract 8 ones, so I will exchange a ten for 10 ones.



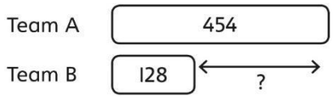


175 – 38 = 137

If the subtraction is a 3-digit number subtract a 2-digit number, children should understand how the recording relates to the place value, and so how to line up the digits correctly.
Children should also understand how to exchange in calculations where there is a zero in the 10s column.

Use bar models to represent subtractions.

'Find the difference' is represented as two bars for comparison.



Bar models can also be used to show that a part must be taken away from the whole.

Children use alternative representations to check calculations and choose efficient methods.

Children use inverse operations to check additions and subtractions.
The part-whole model supports understanding.

I have completed this subtraction.
525 − 270 = 255
I will check using addition.

Column Subtraction
I exchange one hundred for ten tens.

I exchange one ten for ten ones.

First, we subtract the ones and exchange one ten for ten ones.

Next, we subtract the tens and exchange one hundred for ten tens

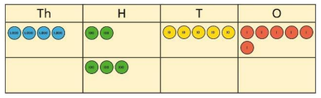
Then we subtract the hundreds.

YEAR 4

- add and subtract numbers with up to 4 digits using the formal written methods of columnar addition and subtraction where appropriate
- solve addition and subtraction two-step problems in contexts, deciding which operations and methods to use and why
- estimate and use inverse operations to check answers to a calculation
- Apply place-value knowledge to known additive and multiplicative number facts (scaling facts by 100)

Use unitising and known facts to support mental calculations.
Make 1,405 from place value equipment.
Add 2,000.
Now add the 1,000s.
1 thousand + 2 thousands = 3 thousands
1,405 + 2,000 = 3,405

Use unitising and known facts to support mental calculations.



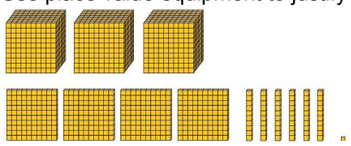
I can add the 100s mentally.
200 + 300 = 500
So, 4,256 + 300 = 4,556

4,256 + 300 = ?

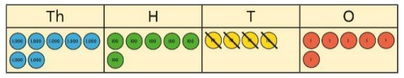
2 + 3 = 5 *200 + 300 = 500*

4,256 + 300 = 4,556

Use place value equipment to justify mental methods.



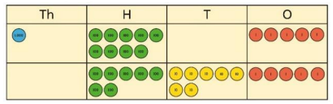
Use place value grids to support mental methods where appropriate.



7,646 − 40 = 7,606

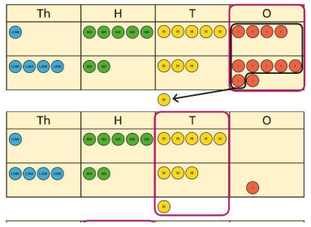
Use knowledge of place value and unitising to subtract mentally where appropriate.
3,501 − 2,000
3 thousands − 2 thousands = 1 thousand
3,501 − 2,000 = 1,501

Ensure that children understand how the columns relate to place value and what to do if the numbers are not all 4-digit numbers.
*Use **equipment**.to show 1,905 + 775.*



Why have only three columns been used for the second row? Why is the Thousands box empty?

Use **images** place value equipment to model required exchanges.



Use a column method to add, including exchanges.

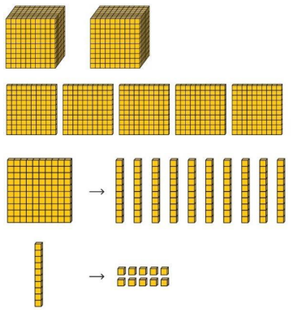
Th	H	T	O
1	5	5	4
+ 4	2	3	7
			1

Th	H	T	O
1	5	5	4
+ 4	2	3	7
	9	1	

Th	H	T	O
1	5	5	4
+ 4	2	3	7
	9	1	

Th	H	T	O
1	5	5	4
+ 4	2	3	7
5	7	9	1

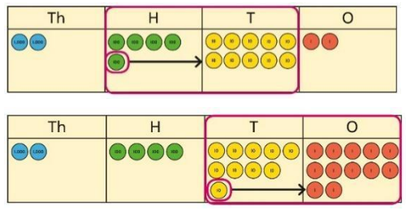
2,502 − 243 = ?



I need to exchange a 10 for some 1s, but there are not any 10s here.

Make exchanges across more than one column where there is a zero as a place holder.

2,502 − 243 = ?



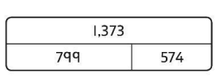
2,502 − 243 = ?

Th	H	T	O
2	4Ø	10	2
−	2	4	3

Th	H	T	O
2	4Ø	9	12
−	2	4	3

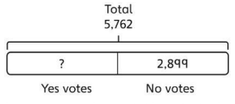
Th	H	T	O
2	4Ø	9	12
−	2	4	3
2	2	5	9

Bar models may be used to represent additions and subtractions in problem contexts, and to justify mental methods where appropriate.

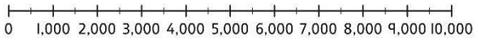


Th	H	T	O
	7	9	9
+	5	7	4
1	3	7	3

I chose to work out 574 + 800, then subtract 1.

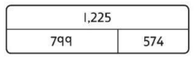


I can work out the total number of Yes votes using 5,762 − 2,899.
Bar models can also represent ‘find the difference’ as a subtraction problem.



912 + 6,149 = ?
I used rounding to work out that the answer should be approximately 1,000 + 6,000 = 7,000.

Use inverse operations to check subtractions.
I calculated 1,225 − 799 = 574.
I will check by adding the parts.



Th	H	T	O
	7	9	9
+	5	7	4
1	3	7	3

The parts do not add to make 1,225.
I must have made a mistake.

partition, place value, tens, hundreds, thousands, column method, whole, part, bar model

Column Addition:
First, we add the ones and carry over.
Next, we add the tens and carry over.
Then we add the hundreds and carry over.
Then we add the thousands together.

Column subtraction
First we subtract the ones.
Next we subtract the tens.
Then we subtract the hundreds.
Then we subtract the thousands.

For exchanging -
I haven’t got enough so I need to go next door and exchange 1 ten/hundred/thousand for 10 ones/tens/hundreds.

Mental Methods
Are the numbers close together? Then count on

YEAR 5

- add and subtract whole numbers with more than 4 digits, including using formal written methods (columnar addition and subtraction)
- solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why
- add and subtract fractions with the same denominator, and denominators that are multiples of the same number

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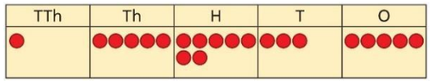
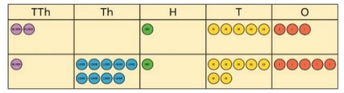
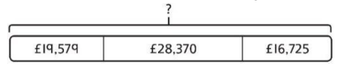
Add a row of counters onto the place value grid to show 15,735 + 4,012.





I need to exchange 10 tens for a 100.

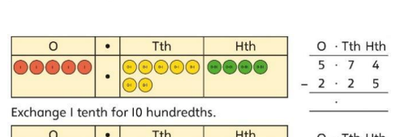
Bar models represent addition of two or more numbers in the context of problem solving.



Use column addition, including exchanges.

TTh	Th	H	T	O
	1	9	1	7	5
+	1	8	4	1	7
	3	7	5	9	2
	1			1	

Two lengths of fencing are 0·6 m and 0·2 m.
How long are they when added together?



0·6 + 0·2 = 0·8
6 tenths + 2 tenths = 8 tenths

$\frac{6}{10} + \frac{2}{10} = \frac{8}{10}$

6 tenths + 2 tenths = 8 tenths
0·6 + 0·2 = 0·8

Show 0·23 + 0·45 using place value counters.

Represent exchange where necessary.

O	·	Tth	Hth
0	·	9	2
+ 0	·	3	3
1	·	2	5

Include examples where the numbers of decimal places are different.

O	·	Tth	Hth
5	·	0	0
+ 1	·	2	5
6	·	2	5

O	·	Tth	Hth
0	·	2	3
+ 0	·	4	5
0	·	6	8

Include exchange where required, alongside an understanding of place value.

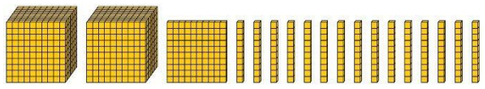
O	·	Tth	Hth
0	·	9	2
+ 0	·	3	3
1	·	2	5

Include additions where the numbers of decimal places are different.

3.4 + 0.65 = ?

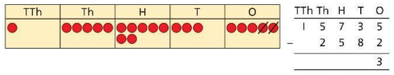
Use place value equipment to understand where exchanges are required.

2,250 – 1,070



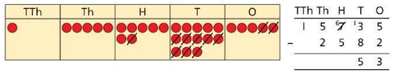
Represent the stages of the calculation using place value equipment on a grid alongside the calculation, including exchanges where required.

15,735 − 2,582 = 13,153



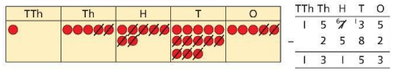
TTh	Th	H	T	O
1	5	7	3	5
−	2	5	8	2
				3

Now subtract the 10s. Exchange 1 hundred for 10 tens.



TTh	Th	H	T	O
1	5	6 7	1 3	5
−	2	5	8	2
			5	3

Subtract the 100s, 1,000s and 10,000s.



TTh	Th	H	T	O
1	5	6 7	1 3	5
−	2	5	8	2
1	3	1	5	3

62,097 − 18,534 = 43,563

TTh	Th	H	T	O
5 6	11 7	1 0	9	7
− 1	8	5	3	4
4	3	5	6	3

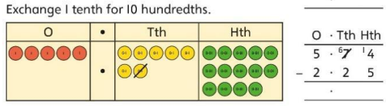
1 m – ☐ m = ☐ m

1 − 0·49 = ?

5·74 − 2·25 = ?

O	·	Tth	Hth
5	·	7	4
− 2	·	2	5
	·		

Exchange 1 tenth for 10 hundredths.

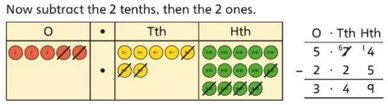


O	·	Tth	Hth
5	·	6 7	1 4
− 2	·	2	5
	·		

Now subtract the 5 hundredths.

O	·	Tth	Hth
5	·	6 7	1 4
− 2	·	2	5
	·		9

Now subtract the 2 tenths, then the 2 ones.



O	·	Tth	Hth
5	·	6 7	1 4
− 2	·	2	5
3	·	4	9

3·921 − 3·75 = ?

O	·	Tth	Hth	Thth
3	·	9	2	1
− 3	·	7	5	0
	·			

---

decimal, column methods, exchange, partition, mental method, ten thousand, hundred thousand

Mental Methods:

Round and Adjust
Because I added too much I need to take it away again
If I add to the addend I need to subtract from the sum

Equivalent
I noticed that one addend was only X away from Y so I moved X from one addend to the other

Partition
When calculating X + Y I chose to partition X into A and B because it is easier to add A to Y then B.

Mental Methods:

Round and Adjust
If I take away too much I have to give it back
If I add to the subtrahend I need to add the same to the difference

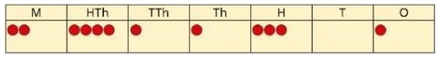
Equivalent
If I add to the subtrahend I have to add to the difference.

Partition
When calculating X - Y I chose to partition X into A and B because it is easier to subtract A from Y then B.

Add instead
I noticed that there was only a small difference between X and Y. So I started counting up from the subtrahend until I reached the

YEAR 6

- perform mental calculations, including with mixed operations and large numbers
- use their knowledge of the order of operations to carry out calculations involving the 4 operations
- solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why
- add and subtract fractions with different denominators and mixed numbers, using the concept of equivalent fractions



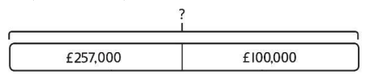
2,411,301 + 500,000 = ?

This would be 5 more counters in the HTh place.

So, the total is 2,911,301.

2,411,301 + 500,000 = 2,911,301

257,000 + 99,000 = ?



I added 100 thousands then subtracted 1 thousand.
257 thousands + 100 thousands = 357 thousands
257,000 + 100,000 = 357,000
357,000 – 1,000 = 356,000
So, 257,000 + 99,000 = 356,000

195,000 + 6,000 = ?

195 + 5 + 1 = 201

195 thousands + 6 thousands = 201 thousands

So, 195,000 + 6,000 = 201,000

decimal, column methods, exchange, partition, mental method, ten thousand, hundred thousand

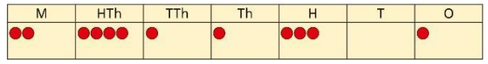
Mental Methods:

Round and Adjust
Because I added too much I need to take it away again
If I add to the addend I need to subtract from the sum

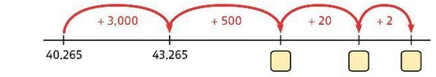
Equivalent
I noticed that one addend was only X away from Y so I moved X from one addend to the other

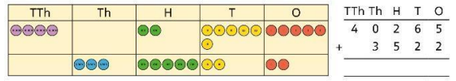
Partition
When calculating X + Y I chose to partition X into A and B because it is easier to add A to Y then B.

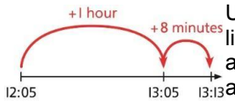
Represent 7-digit numbers on a place value grid, and use this to support thinking and mental methods.



Discuss similarities and differences between methods, and choose efficient methods based on the specific calculation.
Compare written and mental methods alongside place value representations.







Use bar model and number line representations to model addition in problem-solving and measure contexts.

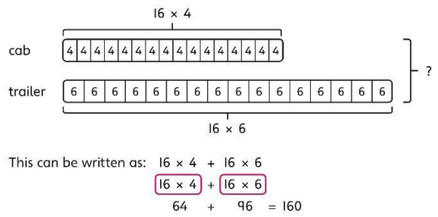
32,145 + 4,302 = ?

TTh	Th	H	T	O
3	2	1	4	5
+	4	3	0	2
3	6	4	4	7

TTh	Th	H	T	O
3	2	1	4	5
+ 4	3	0	2	
7	5	1	6	5

Which method has been completed accurately?
Column methods are also used for decimal additions where mental methods are not efficient.

H	T	O	·	Tth	Hth
1	4	0	·	0	9
+	4	9	·	8	9
1	8	9	·	9	8



Understand the correct order of operations in calculations without brackets.
Understand how brackets affect the order of operations in a calculation.
4 + 6 × 16
4 + 96 = 100

(4 + 6) × 16
10 × 16 = 160

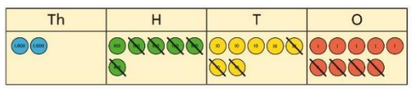
Mental Methods:

Round and Adjust
If I take away too much I have to give it back
If I add to the subtrahend I need to add the same to the difference

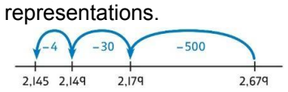
Equivalent
If I add to the subtrahend I have to add to the difference.

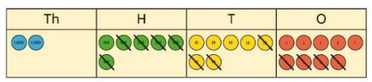
Partition
When calculating X - Y I chose to partition X into A and B because it is easier to subtract A from Y then B.

Add instead
I noticed that there was only a small difference between X and Y. So I started counting up from the subtrahend until I reached the



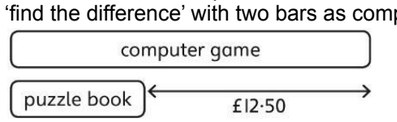
Compare subtraction methods alongside place value representations.





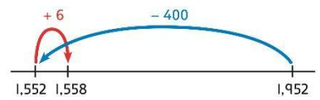
Th	H	T	O
2	6	7	9
−	5	3	4
2	1	4	5

Use a bar model to represent calculations, including 'find the difference' with two bars as comparison.



Compare and select methods.
Use column subtraction when mental methods are not efficient.
Use two different methods for one calculation as a checking strategy.

Th	H	T	O
1	8 ~~4~~	14 ~~5~~	12
− 1	5	5	8
	3	9	4



Use column subtraction for decimal problems, including in the context of measure.

H	T	O	·	Tth	Hth
3	0	9	·	6	0
− 2	0	6	·	4	0
1	0	3	·	2	0

EYFS					whole, part, ones, ten, tens, number bond, add, addition, plus, total, altogether, subtract, subtraction, find the difference, take away, minus, less, more, group, share, equal, **Addition** is putting the parts together. That makes the whole**.** **Subtraction** is taking away a part from the whole. That leaves the other part.